Application of Extended JA Model with Compressive Stress to Thin Plate.

Kyyoul Yun and Keisuke Fujisaki
Toyota Technological Institute
2-12-1, Hisakata, Tempaku, Nagoya, 468-8511, Japan
i71102@toyota-ti.ac.jp

Abstract — Jiles-Atherton's model considered stress dependence is applied to thin plate and compared with the measured hysteresis corves of silicon steel. Magnetic properties, which are important for the precise motor simulation, are affected by compressive stress. An extended Jiles-Atherton's model is found to well describe the measured magnetization curves of silicon steel for small compressive stress, though it is difficult to express the magnetization property with large compressive stress.

I. INTRODUCTION

These days, there are so many electrical devices utilizing various magnetization properties and magnetoelastic couplings of ferromagnetic materials. Magneto-elastic couplings are applied to motors, sensors and actuators, and motors with compressive stress have a large iron loss. So relations between magnetic hysteresis and stress are needed to research for efficiency of motor.

Macroscopic model of hysteretic nonlinear magnetization curves, which is one of the factor of magnetoelastic coupling, were proposed. Preisach considered the existence probability of small magnetic domains having rectangular hysteresis loops [1]. Jiles and Atherton [2] proposed a model of magnetization curve by considering the energy dissipation associated with pinging effect of domain wall movement. The theory was extended by Matsumoto [4] to include mechanical nonlinearity and magnetic hysteresis with stress dependence. Extended magnetic hysteresis with stress dependence is from Jiles and Atherton (JA) equation.

Affection of tensile stress for *B-H* curves of Fe-Ni bulk alloy is studied by comparison extended JA equation with experimental *B-H* curves [5]. Used specimen has 10 mm of thickness. However, the materials used in the electrical motor have around 0.35 mm of thickness. There is a difference between mechanical stress distribution of 3-dimension and 2-dimension, and the magnetoelastic coupling is also considered. So *B-H* curves comparison extended JA equation with experimental data under compressive stress is needed.

In this paper, *B-H* curves comparison of silicon steel with the extended JA model under compressive stress is studied.

II. EXPANDED JA EQUATION

We employ Jiles and Atherton's model [2] extended to stress dependence. That is, the magnetization curve at fixed stress is governed by the next differential equation:

$$\frac{dM}{dH} = \frac{1}{(1+c_0)} \frac{M_{an0} - M}{\delta k_0/\mu_0 - \alpha_0 (M_{an0} - M)} + \frac{c_0}{(1+c_0)} \frac{dM_{an0}}{dH} \quad (1)$$

where M is magnetization, H is magnetic field, μ_0 is space permeability, c, k and α is model parameters [2]. Subscript 0 means initial parameter without stress except μ_0. Function M_{an} denotes the anhysteretic magnetization curve defined by a Langevin function:

$$M_{an0} = M_{s0} L\left(\frac{H_{e0}}{a_0}\right) = M_{s0}\left[\coth\left(\frac{H_{e0}}{a_0}\right) - \frac{a_0}{H_{e0}}\right] \quad (2)$$

In order to take into account the stress dependence of the magnetization curve, we assume that the constitutive parameters in the above model related with the domain structure, a, α and M_s depend on the stress T as follows [6];

$$M_s = M_{s0} + M_{s1}T, \quad \alpha = \alpha_0 + \alpha_1 T, \quad a = a_0 + a_1 T \quad (3)$$

where M_{s1}, a_1 and α_1 are stress dependence constant. In other words, equation (4) is able to be expanded for expressing relation between magnetic field and magnetization under stress as follow;

$$\frac{dM}{dH} = \frac{1}{(1+c_0)} \frac{M_{an} - M}{\delta k_0/\mu_0 - \alpha (M_{an} - M)} + \frac{c_0}{(1+c_0)} \frac{dM_{an}}{dH} \quad (4)$$

Function M_{an} is defined by a Langevin function;

$$M_{an} = M_s L\left(\frac{H_e}{a}\right) = M_s\left[\coth\left(\frac{H_e}{a}\right) - \frac{a}{H_e}\right] \quad (5)$$

where $H_e = H + \alpha M$. Parameters of equation (5) are decided by experimental data. Equations (4) and (5) are possible to express magnetization curve under stress. Magnetization is decided by function of magnetic field related any magnetization process under increasing or decreasing magnetic field. It means that parameters of equations (4) and (5) are decided by measurement of magnetization curve under any stress.

III. PARAMETERS PREPARATION

To evaluate the extended JA equation, it is compared with experiment measurement data. The experimental data of *B-H* curves with compressive stress are used in [6], where 35A360 is used for the silicon steel with compressive stress of 25, 50 and 100 MPa. And *B-H* curve without compressive stress is used in [7].

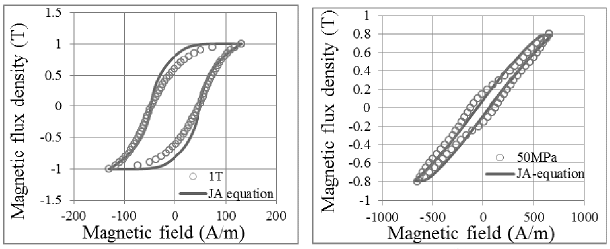
To compare them, 2 steps are used here. The first step is to fit the measured *B-H* curve with the extended JA equation, and to decide the parameters of (1) and (2) for no stress and (4), (5) for compressive stress in order to fit the extended JA equation with the measured hysteresis curves.

The fitted *B-H* curves are shown in Fig. 1. (a) is hysteresis curve at 1 T without compressive stress and (b) is hysteresis curve at 0.8 T with 50 MPa compressive stress. Fitted material parameters are shown in Table 1. A minor loop without compressive stress at 0.8 is calculated using JA parameters of hysteresis curve at 1T, because hysteresis curve without compressive stress at 0.8 is difficult to get data from [6].

The second step is to estimate the parameters using linear approximate equation (3), and to compare hysteresis curves. From Table I and (3), the estimated parameters of compressive stress of 25 MPa and 100 MPa are shown in Table II. Using the parameters as Table II, the hysteresis curves with compressive stress of 25 MPa and 100 MPa are shown in Fig. 2, where the calculation by the extended JA equation and the measurement are shown.

TABLE I
FITTED MATERIAL PARAMETER OF EACH COMPRESSIVE STRESS AS FIG. 1.

	0 MPa	50MPa
M_s	2.999E6 (T)	2.118E6 (T)
α	1.536E-3 (A.U)	1.436E-3 (A.U)
a	1.5655E3 (A/m)	1.6655E3 (A/m)
k_0	65 (A/m)	65 (A/m)
c_0	0.2 (A.U)	0.2 (A.U)



(a) (b)

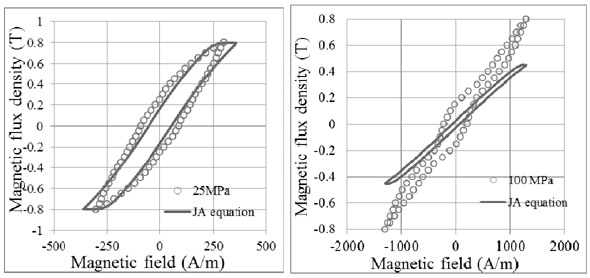
Fig. 1. Comparison between JA model with fitted material parameter of each compressive stress and experimental data (a) Hysteresis comparison of 35A360 (1T hysteresis data of catalog) with JA equation using the 0 MPa parameter of table I (b) Hysteresis comparison of 35A360 (50 MPa compressive stress) with JA equation using the 50 MPa parameter of table I

IV. RESULT AND DISCUSSION

Figure 2 shows comparison of estimated hysteresis using extended JA parameters of Table II with experimental data [6]. Figure 2(a) shows *B-H* curves with 25 MPa compressive stress. Extended JA equation describe *B-H* curve with 25 MPa compressive well. Figure 2(b) shows *B-H* curves with 100 MPa compressive stress. *B-H* curve from Extended JA equation is distinct from experimental B-H curve with 100 MPa compressive stress. It is difficult to express the shape of *B-H* curve using JA equation. JA equation should be developed to explain *B-H* curve of Fig. 2(b).

TABLE II
ESTIMATED MATERIAL PARAMETER OF COMPRESSIVE STRESS AS FIG. 2.

	25 MPa	100 MPa
M_s	2.559E6 (T)	1.237E6 (T)
α	1.486E-3 (A.U)	1.336E-3 (A.U)
a	1.6155E3 (A/m)	1.7655E3 (A/m)
k_0	65 (A/m)	65 (A/m)
c_0	0.2 (A.U)	0.2 (A.U)



(a) (b)

Fig. 2. Comparison of estimated hysteresis using JA equation parameters of table II with experimental data. (a) 25 MPa compressive stress (b) 100 MPa compressive stress

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